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METHODS AND SYSTEMS FOR ANALYZING FLUTTER TEST DATA USING NON-LINEAR TRANSFER FUNCTION FREQUENCY RESPONSE FITTING
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Sheet 1 of 17
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"REPLACEMENT SHEET"

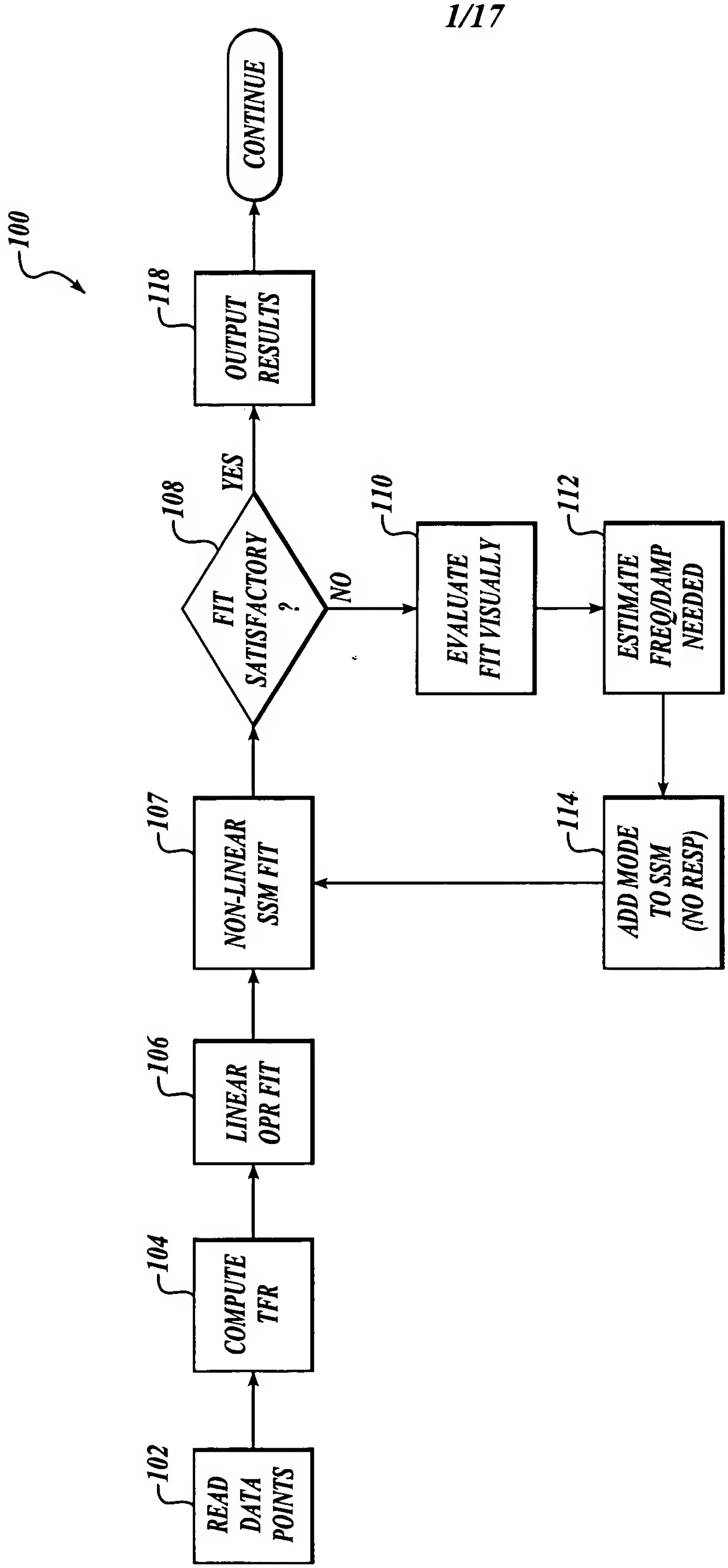
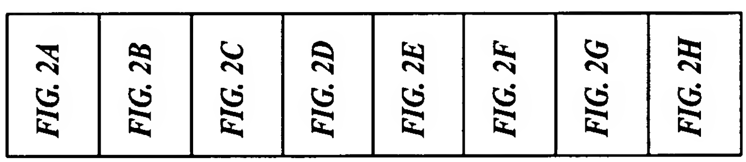


FIG. 1

FIG. 2



$$\frac{\partial \mathrm{Gain}}{\partial x} = \frac{\partial\ (\ W/N_g\ 20.0\ \log_{10}(|Z|)\ )}{\partial x} \tag{2-1}$$

$$\frac{\partial \mathrm{Phase}}{\partial x} = \frac{\partial\ (\ W/N_p\ (180.0/\pi)\ \tan^{-1}(\mathrm{Im}(Z)/\mathrm{Re}(Z))\ )}{\partial x} \tag{2-2}$$

Where:
- Gain = gain of transfer function response in dB
- Phase = phase of transfer function response in degrees
- W = frequency dependent weighting
- Ng = gain normalization
- Np = phase normalization
- Z = complex transfer function frequency response
- x = design variable

Since: $|Z| = \sqrt{Z\ Z^*}$

$$\log_{10}(u) = \log_{10}(e)\ln(u)$$

Gives: $20.0\log_{10}(|Z|) = 10.0\log_{10}(e)\ln(Z\ Z^*)$

Then: $$\frac{\partial \mathrm{Gain}}{\partial x} = \frac{\partial\ (\ W/Ng = 10.0\log_{10}(e)\ \ \ln(Z\ Z^*)}{\partial x}$$

FIG. 2A

Since:

$$\frac{\partial \ln(u)}{\partial x} = \frac{1.0}{u}\frac{\partial u}{\partial x}$$

$$\frac{\partial \tan^{-1}(u)}{\partial x} = \frac{1.0}{1.0+u^2}\frac{\partial u}{\partial x}$$

Then:

$$\frac{\partial \mathrm{Gain}}{\partial x} = \frac{W\ 10.0\log_{10}(e)}{N_g\ (\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\ \frac{\partial(\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}{\partial x}$$

$$\frac{\partial \mathrm{Phase}}{\partial x} = \frac{W\ (180.0/\pi)\ \mathrm{Re}(Z)^2}{N_p\ (\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\ \frac{\partial(\mathrm{Im}(Z)/\mathrm{Re}(Z))}{\partial x}$$

Since:

$$\frac{\partial(u/v)}{\partial x} = \frac{1.0}{v^2}\left(v\frac{\partial u}{\partial x} - u\frac{\partial v}{\partial x}\right)$$

Gives:

$$\frac{\partial \mathrm{Gain}}{\partial x} = \frac{W\ 20.0\log_{10}(e)}{N_p\ (\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\left(\mathrm{Re}(Z)\frac{\partial \mathrm{Re}(Z)}{\partial x} + \mathrm{Im}(Z)\frac{\partial \mathrm{Im}(Z)}{\partial x}\right)$$

$$\frac{\partial \mathrm{Phase}}{\partial x} = \frac{W\ (180.0/\pi)}{N_p\ (\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\left(\mathrm{Re}(Z)\frac{\partial \mathrm{Im}(Z)}{\partial x} + \mathrm{Im}(Z)\frac{\partial \mathrm{Re}(Z)}{\partial x}\right)$$

FIG. 2B

There is similarity between the partial of the gain of the response and that of the phase. To uncover the similarity, examine Equation (2-3):

$$\frac{1.0}{z}\frac{\partial z}{\partial x} = \frac{1.0}{\mathrm{Re}(Z)+\mathrm{Im}(z)j}\left(\frac{\partial \mathrm{Re}(Z)}{\partial x} + \frac{\partial \mathrm{Im}(Z)}{\partial x}j\right) \tag{2-3}$$

Gives:

$$\frac{1.0}{z}\frac{\partial z}{\partial x} = \frac{1.0}{(\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\left(\mathrm{Re}(Z)\frac{\partial \mathrm{Re}(Z)}{\partial x} + \mathrm{Im}(Z)\frac{\partial \mathrm{Im}(Z)}{\partial x}\right) + \frac{1.0}{(\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\left(\mathrm{Re}(Z)\frac{\partial \mathrm{Im}(Z)}{\partial x} - \mathrm{Im}(Z)\frac{\partial \mathrm{Re}(Z)}{\partial x}\right)j$$

Combining the results from Equations (2-1), (2-2) and (2-3) yield Equations (2-4) and (2-5):

$$\frac{\partial \mathrm{Gain}}{\partial x} = \frac{W\ 20.0\log_{10}(e)}{N_g}\,\mathrm{Re}\left(\frac{1.0}{Z}\frac{\partial Z}{\partial x}\right) \tag{2-4}$$

$$\frac{\partial \mathrm{Phase}}{\partial x} = \frac{W\ (180.0/\pi)}{N_p}\,\mathrm{Im}\left(\frac{1.0}{Z}\frac{\partial Z}{\partial x}\right) \tag{2-5}$$

FIG. 2C

The complex response of the block diagonal SSM for a specific transfer function is given by Equation (2-6):

$$z_{ij} = \Sigma \left(\frac{N_{ij}{}^{1}}{D^{1}} \right) + d_{ij} \tag{2-6}$$

Where: $N_{ij}{}^{1} = (c_{i1}{}^{1} b_{1j}{}^{1} + c_{i2}{}^{1} b_{2j}{}^{1}) s + (c_{i2}{}^{1} b_{1j}{}^{1} a_{21}{}^{1} - c_{i1}{}^{1} b_{1j}{}^{1} a_{22}{}^{1} + c_{i1}{}^{1} b_{2j}{}^{1})$

$D^{1} = s^{2} - a_{22}{}^{1} s - a_{21}{}^{1}$

For elements in the D matrix the unknown term in Equations (2-4) and (2-5) is given by Equation (2-7) using Equation (2-6):

$$\frac{\partial z_{ij}}{\partial d_{ij}} = 1.0 \tag{2-7}$$

For elements in the A, B or C matrices, x1, the unknown term in Equations (2-4) and (2-5) is given by Equation (2-8):

$$\frac{\partial z_{ij}}{\partial x^{1}} = \frac{\partial \, (N_{ij}{}^{1} / D_{1})}{\partial x^{1}} \tag{2-8}$$

FIG. 2D

Since: $$\frac{\partial (u/v)}{\partial x} = \frac{1.0}{v^2}\left(v \frac{\partial u}{\partial x} - u \frac{\partial v}{\partial x} \right)$$

Then: $$\frac{\partial z_{ij}}{\partial x^1} = \frac{1.0}{D^1 D^1}\left(D^1 \frac{\partial N_{ij}{}^1}{\partial x^1} - N_{ij}{}^1 \frac{\partial D^1}{\partial x^1} \right)$$

And thus: $$\frac{\partial D^1}{\partial c_{i1}{}^1} = \frac{\partial D^1}{\partial c_{i2}{}^1} = \frac{\partial D^1}{\partial b_{1j}{}^1} = \frac{\partial D^1}{\partial b_{2j}{}^1} = 0.0$$

Simplified: $$\frac{\partial z_{ij}}{\partial x_1} = \frac{1.0}{D^1}\left(\frac{\partial N_{ij}{}^1}{\partial x^1} \right) \quad \text{for } x^1 = c_{i1}{}^1, c_{i2}{}^1, b_{1j}{}^1, b_{2j}{}^1$$

From Equation (2-6) the non-zero partials of the block numerator and denominator are given as Equations (2-9) through (2-16):

$$\frac{\partial N_{ij}{}^1}{\partial c_{i1}{}^1} = b_{1j}{}^1\ s + (\ b_{2j}{}^1 - b_{1j}{}^1\ a_{22}{}^1\ ) \qquad (2\text{-}9)$$

$$\frac{\partial N_{ij}{}^1}{\partial c_{i2}{}^1} = b_{2j}{}^1\ s + (\ b_{1j}{}^1\ a_{21}{}^1\ ) \qquad (2\text{-}10)$$

FIG. 2E

$$\frac{\partial N_{ij}^{1}}{\partial b_{ij}^{1}} = c_{i1}^{1}\ s + (c_{i2}^{1}\ a_{21}^{1} - c_{i1}^{1}\ a_{22}^{1}) \tag{2-11}$$

$$\frac{\partial N_{ij}^{1}}{\partial b_{2j}^{1}} = c_{i2}^{1}\ s + (c_{i1}^{1}) \tag{2-12}$$

$$\frac{\partial N_{ij}^{1}}{\partial a_{21}^{1}} = c_{i2}^{1}\ b_{1j}^{1} \tag{2-13}$$

$$\frac{\partial N_{ij}^{1}}{\partial a_{22}^{1}} = -\ c_{i1}^{1}\ b_{1j}^{1} \tag{2-14}$$

$$\frac{\partial D^{1}}{\partial a_{21}^{1}} = -\ 1.0 \tag{2-15}$$

$$\frac{\partial D^{1}}{\partial a_{22}^{1}} = -\ s \tag{2-16}$$

FIG. 2F

To summarize from Equations (2-4) through (2-16) as Equations (2-17) through (2-30):

$$\frac{\partial Gain_{ij}}{\partial a_{21}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \mathrm{Re}\left(\frac{D^1 c_{i2}{}^1 b_{1j}{}^1 + N_{ij}{}^1}{D^1 D^1 Z_{ij}}\right) \tag{2-17}$$

$$\frac{\partial Phase_{ij}}{\partial a_{21}{}^1} = \frac{W\ (180.0/\pi)}{N_{pij}} \mathrm{Im}\left(\frac{D^1 c_{i2}{}^1 b_{1j}{}^1 + N_{ij}{}^1}{D^1 D^1 Z_{ij}}\right) \tag{2-18}$$

$$\frac{\partial Gain_{ij}}{\partial a_{22}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \mathrm{Re}\left(\frac{-D^1 c_{i1}{}^1 b_{1j}{}^1 + N_{ij}{}^1 s}{D^1 D^1 Z_{ij}}\right) \tag{2-19}$$

$$\frac{\partial Phase_{ij}}{\partial a_{22}{}^1} = \frac{W\ (180.0/\pi)}{N_{pij}} \mathrm{Im}\left(\frac{-D^1 c_{i1}{}^1 b_{1j}{}^1 + N_{ij}{}^1 s}{D^1 D^1 Z_{ij}}\right) \tag{2-20}$$

$$\frac{\partial Gain_{ij}}{\partial b_{1j}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \mathrm{Re}\left(\frac{c_{i1}{}^1 s + c_{i2}{}^1 a_{21}{}^1 - c_{i1}{}^1 a_{22}{}^1}{D^1 Z_{ij}}\right) \tag{2-21}$$

$$\frac{\partial Phase_{ij}}{\partial b_{1j}{}^1} = \frac{W\ (180.0/\pi)}{N_{pij}} \mathrm{Im}\left(\frac{c_{i1}{}^1 s + c_{i2}{}^1 a_{21}{}^1 - c_{i1}{}^1 a_{22}{}^1}{D^1 Z_{ij}}\right) \tag{2-22}$$

$$\frac{\partial Gain_{ij}}{\partial b_{2j}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}} \mathrm{Re}\left(\frac{c_{i2}{}^1 s + c_{i1}{}^1}{D^1 Z_{ij}}\right) \tag{2-23}$$

FIG. 2G

$$\frac{\partial Phase_{ij}}{\partial b_{2j}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{c_{i2}^{1}s+c_{i1}^{1}}{D^{1}Z_{ij}}\right) \tag{2-24}$$

$$\frac{\partial Gain_{ij}}{\partial c_{i1}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{b_{1j}^{1}s+b_{2j}^{1}-b_{1j}^{1}a_{22}^{1}}{D^{1}Z_{ij}}\right) \tag{2-25}$$

$$\frac{\partial Phase_{ij}}{\partial c_{i1}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{b_{1j}^{1}s+b_{2j}^{1}-b_{1j}^{1}a_{22}^{1}}{D^{1}Z_{ij}}\right) \tag{2-26}$$

$$\frac{\partial Gain_{ij}}{\partial c_{i2}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{b_{2j}^{1}s+b_{1j}^{1}a_{21}^{1}}{D^{1}Z_{ij}}\right) \tag{2-27}$$

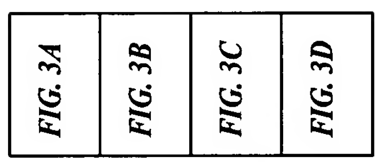
$$\frac{\partial Phase_{ij}}{\partial c_{i2}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{b_{2j}^{1}s+b_{1j}^{1}a_{21}^{1}}{D^{1}Z_{ij}}\right) \tag{2-28}$$

$$\frac{\partial Gain_{ij}}{\partial d_{ij}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{1.0}{Z_{ij}}\right) \tag{2-29}$$

$$\frac{\partial Phase_{ij}}{\partial d_{ij}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{1.0}{Z_{ij}}\right) \tag{2-30}$$

FIG. 2H

FIG. 3



The complex response of the PZM is given by Equation (3-1):

$$Z = \frac{N}{D} = \frac{TFG\ \Pi\ N^1}{\Pi\ D^1} \qquad (3\text{-}1)$$

Where: $N^1 = s^2 + b_1{}^1\, s + b_0{}^1$

$D^1 = s^2 + a_1{}^1\, s + a_0{}^1$

The unknown term in Equations (2-4) and (2-5) is given by Equations (3-2) by using Equation (3-1):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{D\ N}\left(D\frac{\partial N}{\partial x} - N\frac{\partial D}{\partial x}\right) \qquad (3\text{-}2)$$

The results of Equation (3-2) when the transfer function gain is the design variable, x, is given by the Equation (3-3):

$$\frac{1.0}{z}\frac{\partial Z}{\partial x} = \frac{1.0}{TFG} \quad \text{when } x = TFG \qquad (3\text{-}3)$$

FIG. 3A

The results of Equation (3-2) when the a numerator block coefficient is the design variable, x, is given by the Equation (3-4):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N}\left(\frac{\partial N}{\partial x}\right) = \frac{1.0}{N^1}\frac{\partial N^1}{\partial x} \quad \text{when } x = b_1{}^1 \text{ or } b_0{}^1 \tag{3-4}$$

Gives: $$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N^1}s \quad \text{when } x = b_1{}^1$$

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N^1} \quad \text{when } x = b_0{}^1$$

The results of Equation (3-2) when the a denominator block coefficient is the design variable, x, is given by the Equation (3-5):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{D}\left(\frac{\partial D}{\partial x}\right) = -\frac{1.0}{D^1}\frac{\partial D^1}{\partial x} \quad \text{when } x = a_1{}^1 \text{ or } a_0{}^1 \tag{3-5}$$

Gives: $$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D^1}s \quad \text{when } x = a_1{}^1$$

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D^1} \quad \text{when } x = a_0{}^1$$

FIG. 3B

To summarize from Equations (2-4), (2-5), (3-3), (3-4) and (3-5) as Equations (3-6) through (3-15):

$$\frac{\partial Gain}{\partial TFG} = \frac{W\ 20.0\ \log_{10}(e)}{N_g} \frac{1.0}{TFG} \qquad (3\text{-}6)$$

$$\frac{\partial Phase}{\partial TFG} = 0.0 \qquad (3\text{-}7)$$

$$\frac{\partial Gain}{\partial {b_1}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g} Re\left(\frac{s}{N^1}\right) \qquad (3\text{-}8)$$

$$\frac{\partial Phase}{\partial {b_1}^1} = \frac{W\ (180.0/\pi)}{N_p} Im\left(\frac{s}{N^1}\right) \qquad (3\text{-}9)$$

$$\frac{\partial Gain}{\partial {b_0}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g} Re\left(\frac{1.0}{N^1}\right) \qquad (3\text{-}10)$$

FIG. 3C

$$\frac{\partial \mathrm{Phase}}{\partial {b_0}^1} = \frac{W\ (180.0/\pi)}{N_p}\ \mathrm{Im}\left(\frac{1.0}{N^1}\right) \tag{3-11}$$

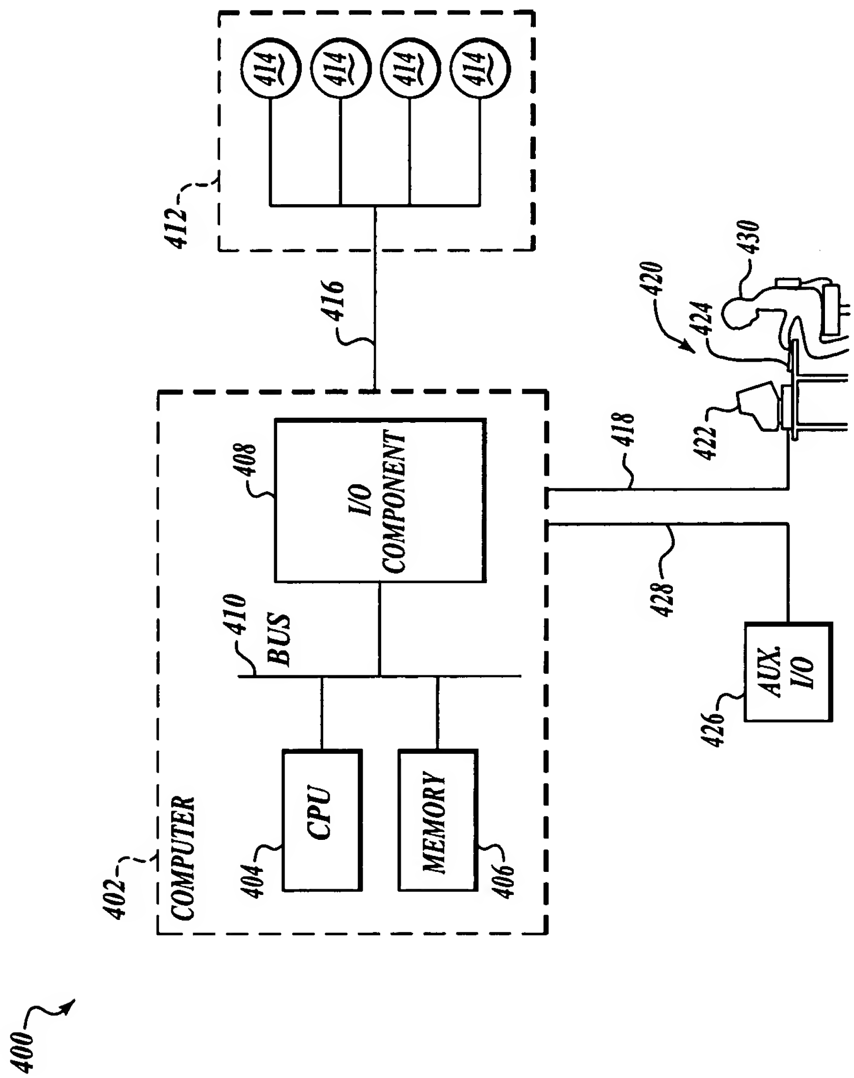
$$\frac{\partial \mathrm{Gain}}{\partial {a_1}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ \mathrm{Re}\left(\frac{-s}{D^1}\right) \tag{3-12}$$

$$\frac{\partial \mathrm{Phase}}{\partial {a_1}^1} = \frac{W\ (180.0/\pi)}{N_p}\ \mathrm{Im}\left(\frac{-s}{D^1}\right) \tag{3-13}$$

$$\frac{\partial \mathrm{Gain}}{\partial {a_0}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ \mathrm{Re}\left(\frac{-1.0}{D^1}\right) \tag{3-14}$$

$$\frac{\partial \mathrm{Phase}}{\partial {a_0}^1} = \frac{W\ (180.0/\pi)}{N_p}\ \mathrm{Im}\left(\frac{-1.0}{D^1}\right) \tag{3-15}$$

FIG. 3D


400
402
COMPUTER
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CPU
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MEMORY
410
BUS
408
I/O COMPONENT
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414
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416
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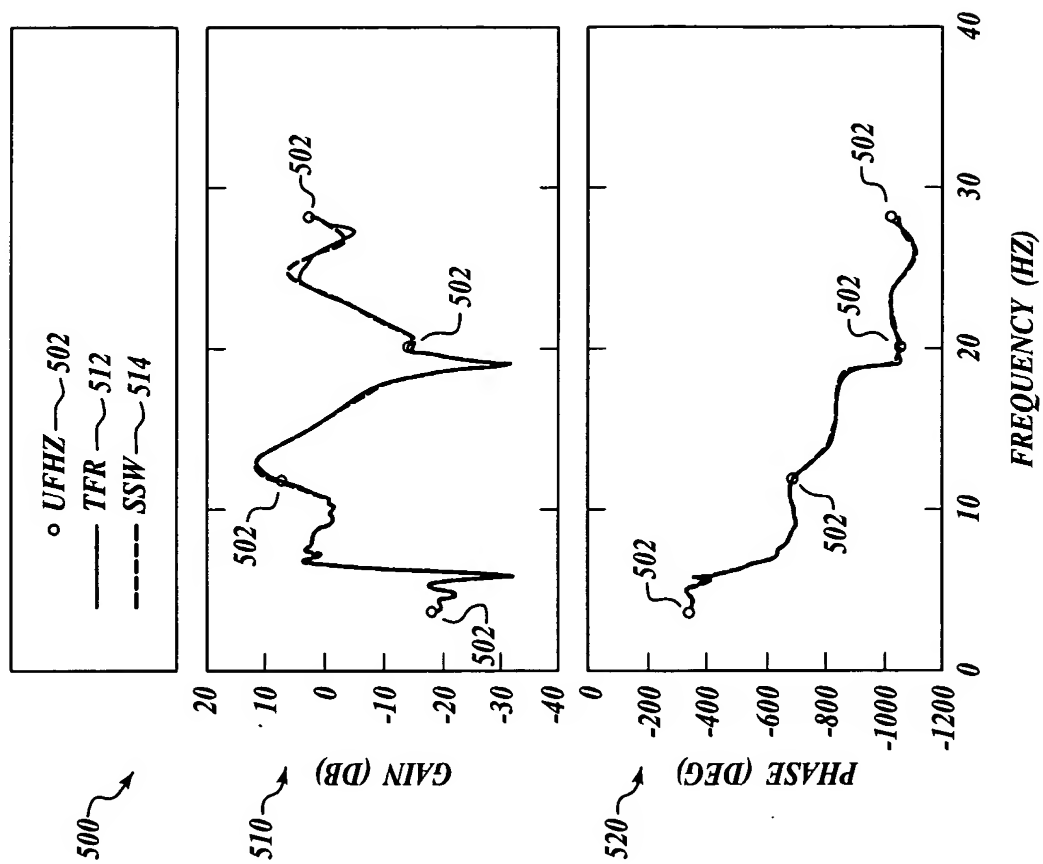
420
422
424
430
426
AUX. I/O
428

FIG. 4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TFR 512
SSW 514
510
GAIN (DB)
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10
0
-10
-20
-30
-40
520
PHASE (DEG)
0
-200
-400
-600
-800
-1000
-1200
502
0
10
20
30
40
FREQUENCY (HZ)

FIG. 5